LOGIC AND THE FIRST PERSON

Primitive notions and the role of the first-person in mathematical models

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Abstract

Here, the author of this thesis will argue that, regardless the field of study, the concept of primitive notion is fundamental to *epistemology*. In fact, along with the *conceptual order*, it provides a way of defining concepts sequentially. Notwithstanding the importance of *primitive notions* in the realm of concepts as well as in *epistemology*, there is no *act of knowing* "something" without an agent-an *epistemic subject*-to assert, that is, to execute the cognitive activities; and this is closely related to the purpose of this thesis, considering that it requires a better understanding of role of the *first-person perspective* in the evaluation and analysis of a mathematical model.

The notion of the order of *conceptual priority* was introduced by Dr. Per Martin-Löf in [2]. In fact, a *concept* precedes another one if the *definition* of the later one is dependent upon the definition of the former. Having defined that, if we draw upon the *epistemic status* of cell activity then we can say that we know that there are specific *molecules* within the cell that catalyze *biochemical reactions* which, in fact, are involved in a variety of cellular processes including *cell growth*, *cell division*, *cell proliferation* and *cell death*. In light of their particular function, those *molecules* actually receive a more sophisticated name, that is, they are known as *enzymes*. The latter *concept*, i.e. being an *enzyme* is solely functional and structural determined.

In order to unveil an entanglement of *notions* paved by the order of *conceptual priority*, we must ask ourselves questions regarding the synthesis of an *enzyme* in the cell environment. Or equivalently, How is an *enzyme* produced in the cell? In fact, if we rely upon the *epistemic status* of the concept of an *enzyme* then we can say that an *enzyme* is a *protein* or a *ribozyme*. Furthermore, the set of enzymes, which are proteins, and the set of enzymes, which are ribozymes, are mutually exclusive. But, what is a *protein*? And, what is a *ribozyme*? Actually, both of them are considered as a *gene-product*. Now, we know that the concept of an *enzyme* is conceptually dependent on the notions of a *protein* and a *ribozyme*, which, in turn, are conceptually dependent on the notion of a *gene*. However, what is a *gene*? Despite the controversy over the concept of a *gene* (see [4] and [5]), we adopt a definition that serves the purpose of our analysis. In fact, according to Gerstein et al [4], a *gene* is a *DNA coding sequence* or a *DNA functional non-coding sequence*. But, the latter concepts are conceptually dependent opon the concept of a *DNA*. So, what is a *DNA*? In fact, a *DNA* is a *double-stranded polymeric macromolecule* that contains *genes* carrying instructions for the whole *life cycle* of a *living organism*.

What is so special about the notion of a *gene* proposed by Gerstein et al in [4]? As we have stated earlier, it is a *circular definition* seeing that it is dependent

upon the notion of a *DNA* whose definition, in turn, draws on the concept of a *gene*. Having said that, one has that the concept of a *gene* seems to be a *primitive notion*, that is, an undefinable notion; which means that it cannot be reduced to a chain of previously well-defined notions in an independent way. What do we mean with "in an independent way"? That is supposed to mean that each element of such a chain has its meaning not referring back to the concept being defined, that is, the concept of a *gene*. Withal, how can we approach such a *metaphysical question*? Or better, how can we understand such a primitive notion then? In order to address this question, we go back to 1884 when Dr. Gottlob Frege started his logicist programme by publishing '*Die Grundlagen der Arithmetik*'. Therewith, he aimed to establish *logic* as a foundation for *arithmetic*, and, consequently, for *mathematics*. Or equivalently, he claimed that *mathematics* was reducible to *logic* by means of *axiomatization*, that is, all the theorems in *arithmetic* could be logically deduced from a set of axioms[1] with perfect accuracy. The latter properties mean *completeness* and *consistency* respectively. So, though conceptually different from one another, one can say that, in *axiomatization*, axioms play the same role as primitive notions. Nonetheless, in 1931, the logicist programme was forestalled by the publications [8, 9] of Dr. Kurt Gödel in which he demonstrated his famous two *incompleteness theorems*. In fact, he thwarted the logicist programme by showing that it is impossible to reduce *mathematics* to a consistent and complete set of *axioms*, seeing that such an *axiomatic system* cannot decide its own *consistency* and *completeness*.

What should we be considering as essential in his attempt to give a *foundation* to *mathematics* with regard to the scope of this thesis? First of all, with the purpose of understanding some phenomenon, one must be able to reduce it to certain concepts that are somehow known so it is not conceivable that we can keep performing a disentanglement of notions forever. So, if we want to assure that we have knowledge of some phenomenon then there must be notions that cannot be reduced to other ones. To quote from Dr. Eyal Shahar in [10]:

> Primitive notions are essential in *epistemology* just as they are essential in *mathematics* and *logic*. They are the building blocks of sequential definitions (...). Without them we do not have the foundation upon which we can state the axioms of science, propose theories, and draw inference.

Withal, how can we get access to the meaning of a primitive notion then? Or equivalently, how can we know a primitive concept? In '*Begriffschrift*' [7], Dr. Gottlob Frege argues that the fundamental *laws of logic*[2] are unprovable. Yet, their *truth* can somehow be known in the sense that one can acknowledge their *truth*. Actually, no mediation is needed for anyone to arrive at their *truth* because it should be an immediate understanding. But, what do we mean with being an immediate understanding? In fact, if we cannot define a primitive notion in terms of previously well-defined concepts then we should be able to clarify its essence, or better, its meaning. Actually, we regard the latter process as a necessary condition to move

[1] An *axiom* is a starting point, i.e. a *premise*; which is not supposed to be conceptually confused with *primitive notion*.
[2] The laws of identity, the excluded middle and the non-contradiction.

through any *entailment of notions* with respect to the *conceptual order*. To quote from Dr. Gottlob Frege in [6]:

> Definitions proper must be distinguished from elucidations *'Erläuterungen'*. In the first stages of a science [*Wenn wir die Wissenschaft beginnen*] we cannot avoid the use of ordinary words [*die Wörter unserer Sprache*]. But these words are, for the most part, not really appropriate for scientific purposes, because they are not precise enough and fluctuate in their use. Science needs technical terms that have precise and fixed *Bedeutungen* , and in order to come to an understanding about these *Bedeutungen* and exclude possible misunderstandings, we provide elucidations. Of course in so doing we have again to use ordinary words, and these may display defects similar to those which the elucidations are intended to remove. So it seems that we shall then have to provide further elucidations. Theoretically one will never really achieve one's goal in this way. In practice, however, we do manage to come to an understanding about the *Bedeutungen* of words. Of course we have to be able to count on a meeting of minds [*ein verständnisvollen Entgegenkommen*], on other's guessing what we have in mind. But all this precedes the construction of a system and does not belong within a system. In constructing a system it must be assumed that the words have precise *Bedeutungen* and that we know what they are. Hence we can at this point leave elucidations out of account and turn our attention to the construction of a system.

So, we can conclude from the latter quotation from Dr. Gottlob Frege that the meaning of primitive notions can only be accessed by *elucidations* [*'Erläuterungen'*], and we presume that it must be driven by *intuition*. But, what kind of intuition then? To answer this question, the author of this thesis would like to quote Dr. Peter Scholze [19]:

> The key issue for me is finding the right definitions; finding the right definitions that really capture the essence of some mathematical phenomenon. I often have some vague vision of what I want to understand, but I am often missing the words to say that.

In his description, one can say that Dr. Peter Scholze is trying to understand some mathematical phenomenon that is solely taking place in his *mind* so he wants to grab it, or rather, he wants to apprehend the essence thereof. That apprehension can subsequently lead him to the formation of a new notion. So, he has knowledge of the mathematical phenomenon without being able to communicate it immediately. That sort of driving force in acquiring knowledge of such a mathematical phenomenon is regarded as an *intellectual intuition* or *non-empirical intuition*. On the other hand, an elucidation of the concept of a *gene*, or better, of the notion of a *gene* is refined in response to an *empirical process*[3] so it is said to be driven by a *sensible intuition*, or better, an *empirical intuition*.

In which sense should we be understanding the latter notions of intuition? How is it essentially connected with the ultimate scope of this thesis? In order to answer

[3] An observation or an experiment.

the former question, we draw upon the *Kantian epistemology* as introduced in [11]. To begin with, as succinctly explained in [13], if we regard a *perspective* as any form of *epistemic access* to "something", that is, an *epistemic object*, then we say that "somebody", or better, an *epistemic subject* has always a *first-person perspective* or *first-person* experience of any *phenomenon* including or related to an *epistemic object*. However, if that access is independent upon the *epistemic subject* then we call it a *third-person perspective*. Now, if "somebody" relies on her own *first-person* experiences so as to understand someone else's *first-person* experience toward an *epistemic object* then we reefer to such an *epistemic access* as a *second-order perspective*. What do we mean with having a *first-person* experience of "something"? In fact, it means to be conscious of "something". But, what is necessary for someone to have a *subjective experience* of "something", or equivalently, to be conscious of "something"[4]? It entails that one perceives herself. Hence, being conscious, or equivalently, having *consciousness* involves the existence of a *self-concept*, a *self-identity* or better, an *ego*. Hence, as for the *Kantian epistemology*, the notion of an *ego* precedes the notion of *consciousness*.

As we have exhaustively argued so far, knowing "something" demands a decomposition of the definition of the concept of that "something" in well-known notions or sufficiently well-understood notions[5]. However, in the latter hypothetical decomposition, each well-known notion has been known at a certain point in *time* and *space* so that knowing the *entailment of notions* leading to the apprehension of that "something" necessitates that one has the ability of unifying them. But, what do we mean with "the ability of unifying them"? In fact, if we agree that the *act of knowing* "something" is preceded[6] by the *act of thinking* which, in turn, is preceded by the *act of being conscious of*[7] what one wants to understand, and, more importantly, if we acknowledge that *consciousness* changes direction in *time* and *space*, that is, we are always conscious of "one thing" at a time, then an *epistemic subject* needs to be endowed with built-in capabilities (*consciousness*, *thinking*, *judgment*, and so forth.) underpinning that unifying process in time and *space*, leading her to have knowledge of "something". The latter conception of a set of built-in capabilities defines *Locke's conception of mind* in [14]. Indeed, Locke's notion of *mind* is a distinct concept in *Kantian epistemology*.

Now, if it is true that manifold *acts* antecede the *act of knowing* according to a *temporal*, a *spacial* or a *conceptual* hierarchy then the *mind* is a multihierarchical set of built-in capabilities. Hence, the set of all built-in sequences of *acts* with respect to that multihierarchy gives rise to the concept of *cognition*. In fact, for instance, an *epistemic subject* perceives an *epistemic object*, that is, "something", at a certain point in *time* and *space*, followed by thinking about the properties of that "something", which, in turn, is followed by *reasoning* through the relation among those properties leading to a *judgment*, that is, the acknowledgement of the

[4]Here, so far, "something" is thought to be an *epistemic object* outside in the world.
[5]In the case of primitive notions.
[6]Here, preceding means going after the other one in *time* and *space*, but not necessarily at the conceptual level. In fact, as we mentioned earlier, the notion of *knowledge* is primitive. However, the *act of knowing* goes after manifold *acts*.
[7]Or equivalently, the *act of perceiving*.

truth of a *claim* as to that "something" which, actually, results in having *knowledge* of that "something". Therefore, one can say that the Lockean *mind* is the set of all built-in capabilities of *cognition*. Moreover, consistently, one has that the concept of *mind* is conceptually dependent on the concept of *cognition* which, in turn, is conceptually dependent on the concept of *consciousness*.

So, what is an *empirical intuition* in the *Kantian epistemology*? It is the acquisition of *knowledge* through experience. In fact, it is grounded in the presumption that the *mind* possesses *a priori* forms of intuition[8] (*space* and *time*, *cause* and *effect*, and so forth) that give form to all the experiences of an *epistemic subject*. To quote from Dr. Immanuel Kant in [12]:

> All our knowledge is thus finally subject to time, the formal condition of inner sense. In it they must all be ordered, connected, and brought into relation.

Clearly, one cannot account for Dr. Peter Scholze's acquisition of *knowledge* on the basis of an *empirical intuition*. Why not? First of all, in his description, the *epistemic object* is in his *mind* by means of an *act of thinking* and is not outside in the world whatsoever. Secondly, being aware of the fact that he is thinking about that "something" entails that he is conscious of his own consciousness which, in turn, demands the acknowledgement of a *self-ego*, or equivalently, an *non-empirical ego*, or better, a *transcendental ego*. In this regard, one has that such an acquisition of *knowledge* describes an *intellectual intuition*. Therefore, the *epistemic subject*, or better, the *judging agent* acknowledging the *truth* of a *thought* or a *claim*, leading to the accretion of *knowledge* can be seen as an *empirical ego* or as a *non-empirical ego*, or equivalently, a *transcendental ego*.

As we have stated in the *abstract* of this thesis, we analyze a *phenomenological model* based on Hill-function type interaction kinetics for *cell differentiation* so as to decide whether or not it adequately defines a *conceptual mechanism* for the performed experiments in [18]. What is a *phenomenological mathematical model*? To answer this question, we refer to the definition given in [3]:

> A traditional definition takes them to be models that only represent observable properties of their targets and refrain from postulating hidden mechanisms and the like.

But, what is essential in the latter definition? In fact, it is crucial to acknowledge the importance of the *modeling agent* who actually translates some *empirical process* into a *mathematical object*[9]. In that translation, she is subject to her own *first-person* experience so as to mathematically describe observable properties of the *target system*[10]. Hence, it means that each component of that *mathematical*

[8] They are not in the world. So, they cannot be known by experience. Actually, as we have argued, they are built in the *mind* so as to enable any form of *consciousness*, or better, any sort of *first-person* experience.

[9] In this thesis, it refers to a system of differential equations.

[10] Or equivalently, the *ontological counterpart*. The latter represents the phenomenon being observed, that is, "Retinoic acid driven mouse embryonic stem cells (mESCs)

object, as well as their relations in that mathematical representation, has an *intentionality*, which, in turn, rightly places our thoughts in the context of *Husserlian philosophy*. To quote from Dr. David W. Smith in [15]:

In Husserl's own words, phenomenology is the science of the essence of consciousness(...). What, briefly, is the essence of consciousness? First, every experience, or act of consciousness, is conscious: the subject experiences it, or is aware of performing it. (...) Second, every act of consciousness is a consciousness of something: in perception I see such-and-such, in imagination I imagine such-and-such, in judgment I judge that such-and-such is the case, and so on. This property of consciousness, its being of or about something, Husserl called intentionality. Thus, we say an experience is intentional, or directed (...) toward some object.

As Dr. Edmund Husserl described in the latter quotation, a subjective experience is directed to an object, that is, it carries an intention. But, how are the *conception order*, the concept of a primitive notion, the concept of judgment and the *first-person perspective* essentially connected with the purpose of this thesis? In fact, *judging* is the activity through which we gain *knowledge*. To quote from Dr. Martin Löf in [2]:

..., namely, to judge is the same as to know, more precisely, to get to know, which is to say that the act of judging is the very act of knowing, and that that which is judged is that which is known, that is, the object of knowledge. And knowing is of course to be taken here as a primitive concept; you can clarify it in various ways, but you cannot reduce it to any other kind of notion.

As one sees in the latter quotation, Dr. Martin Löf identifies the notion of *knowledge* with the notion of *judgment*. So, the acquisition of knowledge depends on the *judging agent*. To quote from Prof. dr. Maria van der Schaar [1]:

Logical questions are independent of psychological questions. But, as a theory of validity of inferences and rationality of our judgements, logic relates to what judging agents do. How can logic be objective if it takes its starting-point in the inferences and judgements we make? Is the judging agent perhaps a transcendental or some other kind of ideal subject?

In order to understand the latter quotation from Prof. dr. Maria van der Schaar, one needs to acknowledge that correct judgements about a *mathematical object* are certainly independent on the *judging agent*. To go further, we draw upon a quotation from Dr. Gottlob Frege in [1]:

judging(acknowledging as true) is certainly an inner mental process; but that something is true is independent of the knowing agent, is objective.

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differentiation in the presence of *PD0325901* or *CHIR99021* or LIF(Lekemia inhibitory factor), or in the absence of *PD0325901* or *CHIR99021* or LIF(Lekemia inhibitory factor)" as reported in [18].

So, if we regard such a *phenomenological model* as a mere *mathematical object* then we need to consider the *Fregean* notion of *judgment*, or equivalently, the logical notion, that is, a *judgment* as an acknowledgement of the *truth* of a claim. In fact, the relation between *judgment* and *truth* is an essential elucidation in Frege's logic.

Drawing upon the *Fregean* notation in [7], one has that

$$\vdash A, \tag{1}$$

should represent a *judgment* that has been made, in which A is the *judgeable content (assertion)*, and the *judgment stroke* $\vdash$ can be interpreted as an *assertive force*, which together with the content A expresses the *judgment*, or better, the *act of judging* which, in turn, by invoking the elucidations of Dr. Martin Löf in [2], is equivalent to the *act of knowing*. To quote from Prof. dr. Maria van der Schaar:

> By putting the judgement stroke in front of an axiom, the agent claims not only that the Thought is true, but that anyone who understands the Thought thereby acknowledges it as true, and is thus entitled to use it as an axiom. By putting the judgement stroke in front of a theorem, the agent claims that anyone who knows the axioms and has made the relevant inference rules evident to himself or herself is entitled to use the theorem as a logical law. These judgements are thus made from a first-person perspective, but they are non-personal at the same time.

So, acknowledging the truth of a mathematical assertion [*theorem*] requires a *first-person* perspective even though the *truth* is independent upon the *judging agent*. Having said that, one can conclude that Dr. Gottlob Frege vehemently believed that *logic* forms the foundation for all the sciences. To quote from Prof. dr. Maria van der Schaar:

> Frege's elucidations of primitive terms differ in an important way from the a priori truths given in the phenomenological tradition. Whereas for Brentano and Husserl descriptive psychology or phenomenology is a science that precedes logic, for Frege logic is the foundational science. Primitive notions, such as judgement and truth, can only be understood by relating them to each other in elucidations. Frege's claim that judging is acknowledging the truth of a Gedanke is such an elucidation. What precedes logic is propaedeutic, consisting of elucidations, sharply to be distinguished from a priori truths, and from definitions as well, which do have a role within logic as science.

So, according to Prof. dr. Maria van der Schaar, elucidations of primitive notions are crucial for anyone who wants to acknowledge the *truth* of a *claim* by herself, but the *truth* of an *assertion* is independent on the *judging agent*. In this regard, the *judging agent* of such a *phenomenological model* can perhaps be understood as a *transcendental ego*, or equivalently, a *non-empirical ego*. However, in [1], Prof. dr. Maria van der Schaar argues that such a *transcendental ego* needed in Frege's logic must differ from the *Kantian transcendental ego*, which, as we introduced earlier, accounts for

self-consciousness. In fact, if a *judgment* in Frege's logic is an acknowledgment of the *truth* of a *claim* then no property is required from such an *ego* needed to assert. Moreover, *via negativa*, as it cannot be the *Kantian empirical-ego* and does not need to be the *Kantian transcendental ego* then she conjectures that such a *transcendental ego* can be regarded as a presupposition so as to enable the constitution of Frege's logic. But, can such a *ego* be conceived? To quote from David Carr in [16]:

...A pure ego [transcendental ego] distinct from the empirical one would seem to be an ego without particular properties. Can such a thing exist?...In order even to think of it as a particular existent, don't we have to think of it as possessing properties?

To these questions a traditional answer has run as follows: What I am conscious of in pure apperception is not a particular, and it is for this reason that I do not need to attribute particular properties to it. Also, for this reason, it must be regarded as distinct from the empirical ego. What I am conscious of is not those particular properties which distinguish me from other persons, but rather those general properties which I share with any and all other egos, such as thinking as such. ...Such an ego is "transcendental" because it transcends all particular egos like you and me.

So, in Dr. David Carr's account of the *Kantian transcendental ego*, as far as the author of this thesis can understand it, such an *ego* is already the one presupposed by Prof. dr. Maria van der Schaar in [1]. In fact, Dr. David Carr argues that being conscious of thinking is not distinguishing, but rather reinforcing the *self-ego* transcendence. Nonetheless, we avoid going in the direction of the *metaphysics* of the *Kantian transcendental ego*[11] given that it deviates from the scope of this section. Actually, the author of this thesis regards such an account as a very difficult task and he doubts whether he would have the tools to do that. In sum, so far, he wants to emphasize that if a *phenomenological model* is merely regarded as a *mathematical object* then a logical notion of *judgment*-an acknowledgement of the *truth* of a thought- is required, considering that the correctness of mathematical judgements is objective. But, there is no assertion without an agent so, in this case, as she cannot be *psychological*, that is, *empirical*, then such a *judging agent* can be understood as a *transcendental ego*.

Now, if we consider that such a *phenomenological model* is inherently *psychological*, or equivalently, if we acknowledge that each element of such a *phenomenological model*, as well as their relation, has a specific *intention*. How do those *intentions* manifest themselves in the *model*? In fact, they are supposed to comply with the *epistemic status* of the *ontological counterpart* and, more importantly, to promote the reproduction of the observations thereof. So, in this regard, we are in need of a psychological notion, or better, an empirical notion of *judgment*. In this regard, a *judgment* is a mental activity in response to an *empirical process*. To quote from Prof. dr. Maria van der Schaar in [1]:

When studying the act of judgement, one may distinguish two different points of view: one may study judgement from an empirical or from a logical point of

[11] See [17] for a thorough approach of the *metaphysics* of the *Kantian self-ego*.

view. From an empirical point of view, one understands judgement as an event in the world, to be represented by a predicate. Describing what John does, one may say 'John judges that snow is white'. Judging is here understood as a relation obtaining between John and the thought that snow is white.

To understand the latter quotation, we can give an example. Indeed, Let

$$\mathcal{M}_{(a_P,a_X,a_E,b,c,d,\theta_X,\theta_E,\theta,k,n)} \tag{2}$$

represent the *phenomenological model* that we shall analyze in this thesis, whereby $\mathcal{M}_{(a_P,a_X,a_E,b,c,d,\theta_X,\theta_E,\theta,k,n)}$ denote the model $\mathcal{M}$ with *parameter setting*

$$(a_P, a_X, a_E, b, c, d, \theta_X, \theta_E, \theta, k, n) \in \mathbb{R}^{11}.$$

Given that each of the latter *parameters* has an *intentionality* [*directionality*], the *judging agent* might claim that

$$\vdash^{\Psi} \quad a_P \gg 1 \rightarrow sc^{\mathcal{M}} \sim \mathcal{O}_{\mathrm{P}}^{(\mathcal{CHIR}^+,\mathcal{PD}^+,\mathcal{LIF}^+,\mathcal{RA}^-)}, \tag{3}$$

which means that choosing the *parameter* a_P much greater than 1 implies that the *model* gives rise to a *scenario* $sc^{\mathcal{M}}$ that is similar [$\sim$] to the *observation*

$$\mathcal{O}_{\mathrm{P}}^{(\mathcal{CHIR}^+,\mathcal{PD}^+,\mathcal{LIF}^+,\mathcal{RA}^-)}. \tag{4}$$

But, what is a *scenario*? Even though the definition of a *scenario* is only provided further in this chapter, one can, so far, say that it is indeed the *mathematical counterpart* of an *observation*, that is, a *description* of some of the model's properties with respect to a specific question. Furthermore, in (3), the sign $\vdash^{\Psi}$ must not be confused with the *Fregean judgment stroke* $\vdash$, which is an assertive force, acknowledging the *truth* [*objective*] of a *thought* [*claim*], but instead, $\vdash^{\Psi}$ ought to be interpreted as a sign representing a *mental activity*[12] that has occurred in *time* and *space* in response to an *empirical process*. Hence, in this case, one has that a empirical notion of *judgment* is the product of a mental process in response to experience so it is an event in the world characterized by a relation between the *thinker*, or better, the *agent* and a *thought*. In fact, asserting that the *parameter* $a_P \gg 1$ should correspond to an *observation*, in our case, is representing a property of the observable (*experiment*). Thereby, as an event in the world, one has that a *judgment* as an empirical phenomenon demands a *third-person perspective*.

Now, what is the main hypothesis in the latter approach leading to the 'construction' of the *scenario* $sc^{\mathcal{M}}_{\lambda_m}$ similar [$\sim$] to the observation $\mathcal{O}$? In fact, it is essential to knowing[13] the *scenario* $sc^{M}_{\lambda_0}$.

[12]Here, Ψ should be regarded as the psychology symbol. In so doing, $\vdash^{\Psi}$ betokens that a *judgment* has been made in response to an empirical process.

[13]As we have defined earlier in this section, knowing a scenario means knowing its properties.

As for the *evaluation* of such a *phenomenological model*, one still needs to decide whether or not a *scenario* is adequately similar [$\sim$] to an *observation*. In fact, owing to the fact that the *judging agent* and the *modelling agent* might not be the same *epistemic subject*, the author of this thesis conjectures that a proper answer for the latter question perhaps requires that one sheds light on the role of the *second-person perspective* in *mathematical models*.

In sum, the author of this thesis has purported to illustrate the *duality* of the *judging agent* as a *transcendental ego* and as a *psychological ego*, or rather, as a *non-empirical ego* and as an *empirical ego* respectively. In fact, in the analysis of such a *phenomenological model*, through which judging actions are made, the *judging agent* can be seen as the sum of two components, that is, two projections of herself entirely necessary for her to keep getting knowledge of such a *model*. Furthermore, this duality must be emphasized in the discrimination of the two involved perspectives, that is, the *first-person perspective* and the *third-person perspective*, along with the difference between a logical and an empirical notion of *judgment*. Ergo, undermining such a duality of the *judging agent* can presumably cause an impingement upon the *evaluation* of a proposed mathematical representation of a *target system*.

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